i) **Faraday's law of Induction:–** Faraday observed that when the magnetic flux threw a conducting circuit changed a current flow in the circuit. this induced current flow in such a way to oppose the original change in m.f. (Lenz's law) the induced current produces its on magnetic field and the direction of flow of current is such that this secondary magnetic field ~~is~~ in the opposite direction to the original magnetic field.

The change in magnetic flux leads to an electromotive force '$\mathcal{E}$' which then produces the observed current

N – no. of lines of force

$$'\mathcal{E}' = -\frac{dN}{dt} \quad \text{——— (1)}$$

in this eqⁿ magnetic flux is represented by the no. of lines of force N, and (–) sign shows the opposition to change.

The induced electromotive force corresponds to the e.f. taken all around the circuit and can be written as

$$\mathcal{E} = \int_s \vec{E} \cdot d\vec{s} \quad \text{——— (2)}$$

eqn (2) Can be written as,

$$\oint_S \vec{E}\cdot d\vec{s} = \int_A \text{curl}\,\vec{E}\cdot d\vec{A} \quad \text{——— (3)}$$

$d\vec{A}$ is an element of the area enclosed by the circuit.

The no. of lines of force N can be related to magnetic field B by the relation-

$$N = \int_A \vec{B}\cdot d\vec{A} \quad \text{——— (4)}$$

On diffestiating both sides of (4)

$$\frac{dN}{dt} = \int_A \frac{\partial \vec{B}}{\partial t}\cdot d\vec{A} \quad \text{——— (5)}$$

(diff. partially Bcoz $\vec{B}$ depend on t as well as position)

by (3), (4) & (5)
eqn (1) Can be re written as,

$$\int_A \text{curl}\,\vec{E}\cdot d\vec{A} = -\int \frac{\partial \vec{B}}{\partial t}\cdot d\vec{A}$$

if we further differentiate, above eqn

$$\text{curl}\,\vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

or, $\boxed{\vec{\nabla}\times\vec{E} + \frac{\partial \vec{B}}{\partial t} = 0}$

This eqn shows that the changing magnetic field induced an electric field

Energy in the Magnetic field

The amount of charge per unit time passing threw the wire is current I. the total work done per unit time is ~~one~~

$$\frac{dW}{dt} = -\overset{e.m.f.}{\vec{\mathcal{E}}} I = LI\frac{dI}{dt} \qquad \left\{ \because \mathcal{E} = -\frac{dI}{dt} \right\} \quad \text{——— (1)}$$

If we start with zero current and build it up to a final value I, (or, integrating this eq^n over times) ~~then~~, work done –

$$\boxed{W = \frac{1}{2} L I^2} \quad \text{——— (2)}$$

We have,

$$(\text{flux})\ N = \int_S \vec{B} \cdot d\vec{s}$$

$$\because \int_S \vec{B} \cdot d\vec{s} = \int_S (\vec{\nabla} \times \vec{A}) \cdot d\vec{s} = \oint_P \vec{A} \cdot d\vec{l}$$

$\hookrightarrow$ parameter

$$\therefore N = \oint_P \vec{A} \cdot d\vec{l}$$

P is parameter of loop and s is surface bounded within P.

then

$$LI = \oint \vec{A} \cdot d\vec{l} \quad \text{——— (3)}$$

using (3) in (2), we get

$$W = \frac{1}{2} I \oint \vec{A} \cdot d\vec{l}$$

$$\text{or,}\quad W = \frac{1}{2} \oint (\vec{A} \cdot \vec{I})\, d\vec{l} \quad \text{——— (4)}$$

On generalizing the volume current, then

$$W = \frac{1}{2} \oint (\vec{A} \cdot \vec{J})\, d\tau \quad \text{——— (5)}$$

and, from Ampere's law:-

$$\vec{\nabla}\times\vec{B} = \mu_0\vec{J} \Rightarrow \vec{J} = \frac{\vec{\nabla}\times\vec{B}}{\mu_0}$$

put in (5)

$$W = \frac{1}{2\mu_0}\int \vec{A}\cdot(\vec{\nabla}\times\vec{B})\,d\tau \quad \text{—— (6)}$$

We have, using vector Identity,

$$\vec{\nabla}\cdot(\vec{A}\times\vec{B}) = \vec{B}\cdot(\vec{\nabla}\times\vec{A}) - \vec{A}\cdot(\vec{\nabla}\times\vec{B})$$

or,

$$\vec{A}\cdot(\vec{\nabla}\times\vec{B}) = \vec{B}\cdot(\vec{\nabla}\times\vec{A}) - \vec{\nabla}\cdot(\vec{A}\times\vec{B})$$

$$\vec{A}\cdot(\vec{\nabla}\times\vec{B}) = \vec{B}\cdot\vec{B} - \vec{\nabla}\cdot(\vec{A}\times\vec{B})$$

Substituting this eqn in (6), finally we get,

$$W = \frac{1}{2\mu_0}\left[\int \vec{B}^2\,d\tau - \int \vec{\nabla}\cdot(\vec{A}\times\vec{B})\,d\tau\right]$$

$$W = \frac{1}{2\mu_0}\left[\int B^2\,d\tau - \oint \vec{A}\times\vec{B}\cdot d\vec{s}\right] \quad \text{—— (7)}$$

If we integrate over all space then, the surface integral term in eqn (7) becomes zero and therefore eqn (7) reduces to:

$$\boxed{W = \frac{1}{2\mu_0}\int B^2\,d\tau} \quad \text{—— (8)}$$

This eqn shows that the energy is hold in the magnetic field is $B^2/2\mu_0$ per unit volume.